Linear Equation in Two Variables

Chapter- 03

Conceptual Notes for NTSE/KVPY/Olympiad/Boards

Table of Contents :

Linear equation in two variables:

A statement of equality of two algebraic expressions, which involve one or more unknown quantities is known as an equation. If there are two unknown quantities then equation is called linear equation in two variables.

- A linear equation is an equation which involves linear polynomials.
- A value of the variable which makes the two sides of the equation equal is called the solution of the equation.
- Same quantity can be added/subtracted to/from both the sides of an equation without changing the equality.
- Both the sides of an equation can be multiplied/divided by the same non-zero number without changing the equality.

Note :- To find value of variables in any equation we required number of equation equal to number of variables in equation.

General form of pair of linear equation:

$$\left.\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned}\right\}$$ where a_1, b_1, c_1 & a_2, b_2, c_2 are constants.

Graph of Linear Equation $ax + by + c = 0$ in two variables, where $a \neq 0, b \neq 0$:

(i) Step I : Obtain the linear equation, let the equation be $ax + by + c = 0$.

(ii) Step II : Express y in terms of x to obtain $y = -\left(\frac{ax+c}{b}\right)$

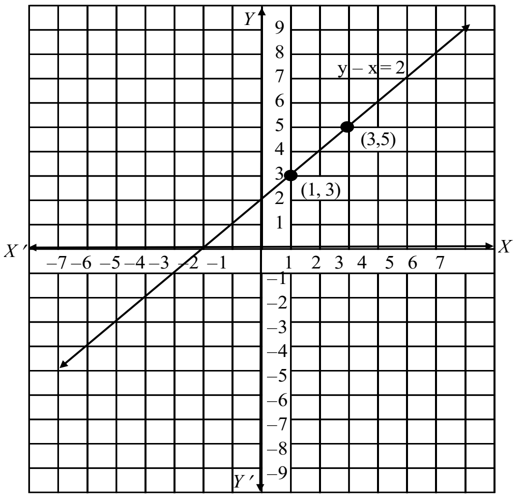
(iii) Step III : Give any two values to x and calculate the corresponding values of y from the expression in step II to obtain two solutions, say (α_1, β_1) and (α_2, β_2). If possible take values of x as integers in such a manner that the corresponding values of y are also integers.

(iv) Step IV : Plot points (α_1, β_1) and (α_2, β_2) on a graph paper.

(v) Step V : Join the points marked in step IV to obtain a line. The line obtained is the graph of the equation $ax + by + c = 0$.

Illustrations

Ex.1 Draw the graph of the equation y – x = 2.



Sol. We have, $y - x = 2 \Rightarrow y = x + 2$

When x = 1, we have : y = 1 + 2 = 3

When x = 3, we have : y = 3 + 2 = 5

Thus, we have the following table exhibiting the abscissa and ordinates of points on the line represented by the given equation.

x	1	3
y	3	5

Plotting the points (1, 3) and (3, 5) on the graph paper and drawing a line joining them, we obtain the graph of the line represented by the given equation as shown in Fig.

Graphical Representation of Pair of Linear Equations :

Let the system of pair of linear equations be

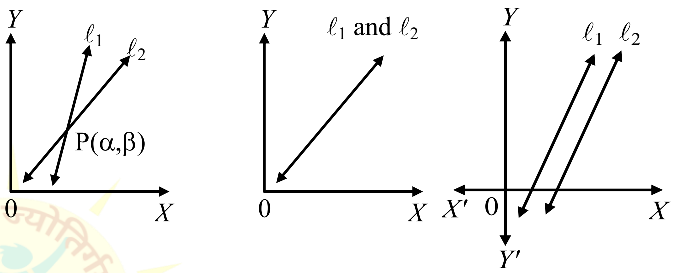
$a_1x + b_1y = c_1$ ....(1) $a_2x + b_2y = c_2$ ....(2)

We know that given two lines in a plane, only one of the following three possibilities can happen -

(i) The two lines will intersect at one point.

(ii) The two lines will not intersect, however far they are extended, i.e., they are parallel.

(iii) The two lines are coincident lines.



Ex.2. The path of highway number 1 is given by the equation x + y = 7 and the highway number 2 is given by the equation 5x + 2y = 20. Represent these equations geometrically.

Sol. We have, $x + y = 7 \Rightarrow y = 7 - x$ ....(1)

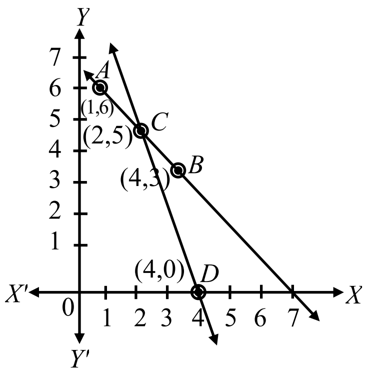
In tabular form :

x	1	4
y	6	3
Points	A	B

And $5x + 2y = 20 \Rightarrow y = \frac{20 - 5x}{2}$ ....(2)

In tabular form :

x	2	4
y	5	0
Points	C	D



Plot the points A (1, 6), B(4, 3) and join them to form a line AB.

Similarly, plot the points C(2, 5). D (4, 0) and join them to get a line CD. Clearly, the two lines intersect at the point C. Now, every point on the line AB gives us a solution of equation (1). Every point on CD gives us a solution of equation (2).

Ex.3 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Sol. Let the number of boys be x and the number of girls be y. Then the equations formed are

x + y = 10 ....(1) and y = x + 4 ....(2)

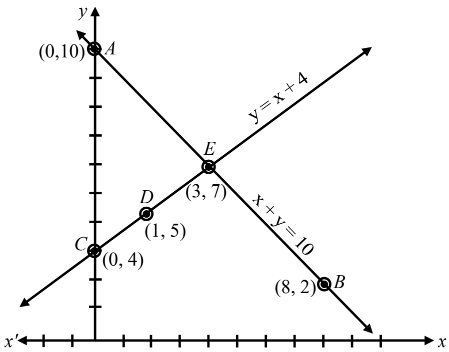
Let us draw the graphs of equations (1) and (2) by finding two solutions for each of the equations. The solutions of the equations are given.

$x + y = 10$

x	0	8
$y = 10 - x$	10	2
Points	A	B

$y = x + 4$

x	0	1	3
$y = x + 4$	4	5	7
Points	C	D	E



i.e. Number of boys = 3

Number of girls = 7.

Plotting these points we draw the lines AB and CE passing

Verification :

through them to represent the equations. The two lines AB and Ce intersect at the point E (3, 7). So, x = 3 and y = 7 is the required solution of the pair of linear equations.

Putting x = 3 and y = 7 in (1), we get
L.H.S. = 3 + 7 = 10 = R.H.S., (1) is verified.
Putting x = 3 and y = 7 in (2), we get
7 = 3 + 4 = 7, (2) is verified.
Hence, both the equations are satisfied.

Type of Solutions : There are three types of solutions :

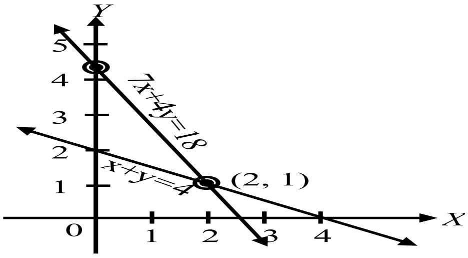
1. **Unique solution.**
2. **Infinitely many solutions**
3. **No solution.**

(A) Consistent : If a system of simultaneous linear equations has at least one solution then the system is said to be consistent.

(i) Consistent equations with unique solution : The graphs of two equations intersect at a unique point.

For example.

Consider $x + 2y = 4$
$7x + 4y = 18$

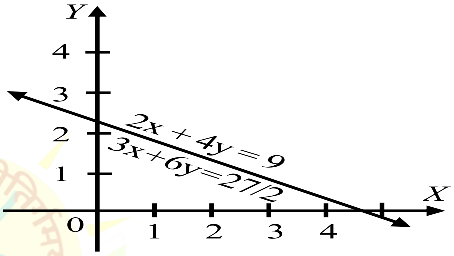


The graphs (lines) of these equations intersect each other at the point (2, 1) i.e., x = 2, y = 1.
Hence, the equations are consistent with unique solution.

(ii) Consistent equations with infinitely many solutions : The graphs (lines) of the two equations will be coincident.

For example. Consider

$$2x + 4y = 9 \Rightarrow 3x + 6y = \frac{27}{2}$$



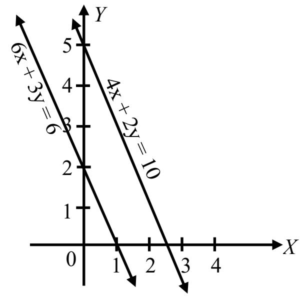
The graphs of the above equations coincide. Coordinates of every point on the lines are the solutions of the equations. Hence, the given equations are consistent with infinitely many solutions.

(B) Inconsistent Equation : If a system of simultaneous linear equations has no solution, then the system is said to be inconsistent.

(C) No Solution : The graph (lines) of the two equations are parallel.

For example. Consider $4x + 2y = 10$ & $6x + 3y = 6$

The graphs (lines) of the given equations are parallel. They will never meet at a point.
So, there is no solution. Hence, the equations are inconsistent.



S.No	Graph of Two Equations	Types of Equations
1	Intersecting lines	Consistent, with unique solution
2	Coincident	Consistent with infinite solutions
3	Parallel lines	Inconsistent (No solution)

Ex.4 Show graphically that the system of equations $x - 4y + 14 = 0$; $3x + 2y - 14 = 0$ is consistent with unique solution

Sol. The given system of equations is

$x - 4y + 14 = 0$ ....(1) $\Rightarrow y = \frac{x+14}{4}$

When $x = 6$, $y = \frac{6+14}{4} = 5$ When $x = -2$, $y = \frac{-2+14}{4} = 3$

In tabular form :

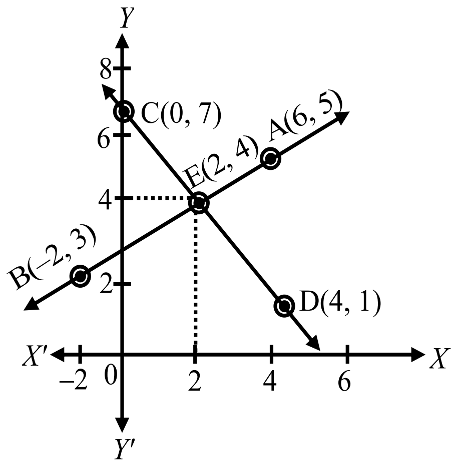
x	6	−2
y	5	3
Points	A	B

$3x + 2y - 14 = 0$ ....(2) $\Rightarrow y = \frac{-3x+14}{2}$

When $x = 0$, $y = \frac{0+14}{2} = 7$ When $x = 4$, $y = \frac{-3\times4+14}{2} = 1$

In tabular form :

x	0	4
y	7	1
Points	C	D



The given equations representing two lines, intersect each other at a unique point (2, 4).
Hence, the eq^n are consistent with unique sol^n.

Ex.5 Show graphically that the system of equations 2x + 3y = 10, 4x + 6y = 12 has no solution.

Sol. The given equations are 2x + 3y = 10

$\Rightarrow \quad 3y = 10 - 2x \qquad \Rightarrow y = \frac{10-2x}{3}$

When x = – 4, $y = \frac{10-2\times(-4)}{3} = \frac{10+8}{3} = 6$

When x = 2, $y = \frac{10-2\times 2}{3} = \frac{10-4}{3} = 2$

In tabular form :

x	– 4	2
y	6	2
Points	A	B

4x + 6y = 12 $\Rightarrow$ 6y = 12 – 4x

$\Rightarrow$ 6y = 12 – 4x $\Rightarrow$ $y = \frac{12-4x}{6}$

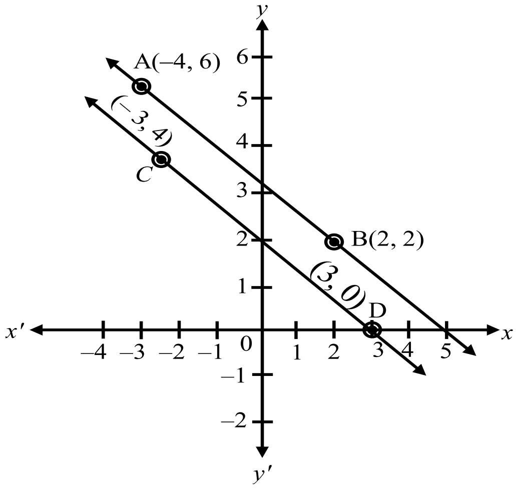
When x = – 3, $y = \frac{12-4\times(-3)}{6} = \frac{12+12}{6} = 4$

When x = 3, $y = \frac{12-4\times(3)}{6} = \frac{12-12}{6} = 0$

In tabular form :

x	– 3	3
y	4	0
Points	C	D

Plot the points A (–4, 6), B(2, 2) and join them to form a line AB. Similarly, plot the points C(–3, 4), D(3, 0) and join them to get a line CD.



Clearly, the graphs of the given equations are parallel lines. As they have no common point, there is no common solution. Hence, the given system of equations has no solution.

Ex.6 Draw the graphs of the following equations ; 2x – 3y = – 6 ; 2x + 3y = 18; y = 2

Find the vertices of the triangles formed and also find the area of the triangle.

Sol. (a) Graph of the equation 2x – 3y = – 6;

We have, 2x – 3y = – 6 $\Rightarrow$ $y = \frac{2x+6}{3}$

When, x = 0, $y = \frac{2\times 0+6}{3} = 2$

When, x = 3, $y = \frac{2\times 3+6}{3} = 4$

Then, we have the following table :

x	0	3
y	2	4

Plotting the points P(0, 2) and Q(3, 4) on the graph paper and drawing a line joining between them we get the graph of the equation 2x – 3y = – 6 as shown in fig.

(b) Graph of the equation 2x + 3y = 18;

We have 2x + 3y = 18 $\Rightarrow$ $y = \frac{-2x+18}{3}$

When, x = 0, $y = \frac{-2\times 0+18}{3} = 6$

When, x = – 3, $y = \frac{-2\times(-3)+18}{3} = 8$

Then, we have the following table :

x	0	– 3
y	6	8

Plotting the points R(0, 6) and S(–3, 8) on the same graph paper and drawing a line joining between them, we get the graph of the equation 2x + 3y = 18 as shown in fig.

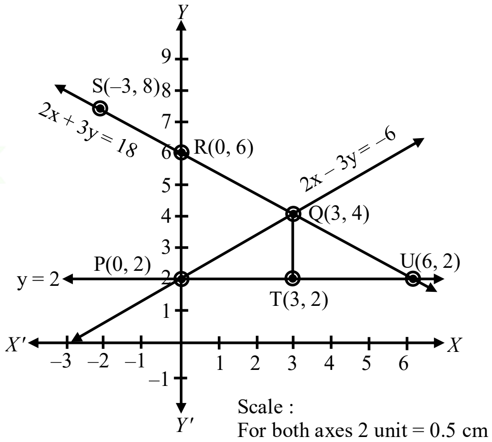
(c) Graph of the equation y = 2

It is a clear fact that y = 2 is for every value of x. We may take the points T (3, 2), U(6, 2) or any other values.

Then, we get the following table :

x	3	6
y	2	2

Plotting the points T(3, 2) and U(6, 2) on the same graph paper and drawing a line joining between them, we get the graph of the equation y = 2 as shown in fig.



From the fig., we can observe that the lines taken in pairs intersect each other at points Q(3, 4), U (6, 2) and P(0, 2). These form the three vertices of the triangle PQU.

To find area of the triangle so formed

The triangle is so formed is PQU (see fig.)

In the ΔPQU ,

QT (altitude) = 2 units

and PU (base) = 6 units

so, area of ΔPQU = $\frac{1}{2}$ (base × height)

$= \frac{1}{2}$ (PU × QT) = $\frac{1}{2} \times 6 \times 2$

= 6 sq. units.

Important Points to be Remembered:

Pair of lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratio
$2x+3y+4=0$ $5x+6y+9=0$	$\frac{2}{5}$	$\frac{3}{6}$	$\frac{4}{9}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$x+2y+5=0$ $3x+6y+15=0$	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{5}{15}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$2x-3y+4=0$ $4x-6y+10=0$	$\frac{2}{4}$	$\frac{-3}{-6}$	$\frac{4}{10}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Graphical representation	Algebraic interpretation
Intersecting lines	Exactly one solution (unique)
Coincident lines	Infinitely many solutions
Parallel lines	No solution

From the table above you can observe that if the line $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

(i)	for the intersecting lines then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
(ii)	for the coincident lines then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
(iii)	for the parallel lines then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Ex.7 On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ and without drawing them, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide.

(i) $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$, $2x - y + 9 = 0$

Sol. Comparing the given equations with standard forms of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ we have,

(i) $a_1 = 5, b_1 = -4, c_1 = 8;$ $a_2 = 7, b_2 = 6, c_2 = -9$ $\therefore \frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6}$ $\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Thus, the lines representing the pair of linear equations are intersecting.

(ii) $a_1 = 9, b_1 = 3, c_1 = 12;$ $a_2 = 18, b_2 = 6, c_2 = 24$ $\therefore \frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Thus, the lines representing the pair of linear equation coincide.

(iii) $a_1 = 6, b_1 = -3, c_1 = 10;$ $a_2 = 2, b_2 = -1, c_2 = 9$ $\therefore \frac{a_1}{a_2} = \frac{6}{2} = 3, \frac{b_1}{b_2} = \frac{-3}{-1} = 3, \frac{c_1}{c_2} = \frac{10}{9}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Thus, the lines representing the pair of linear equations are parallel.

Algebraic Solution of a System of Linear Equations:

Sometimes, graphical method does not give an accurate answer. While reading the coordinates of a point on a graph paper, we are likely to make an error. So, we require some precise method to obtain accurate result. Algebraic methods given below yield accurate answers.

(i) Method of elimination by substitution.
(ii) Method of elimination by equating the coefficients.
(iii) Method of cross multiplication.

➢ **Substitution Method :** In this method, we first find the value of one variable (y) in terms of another variable (x) from one equation. Substitute this value of y in the second equation. Second equation becomes a linear equation in x only and it can be solved for x.

- Putting the value of x in the first equation, we can find the value of y.
- This method of solving a system of linear equations is known as the method of elimination by substitution.
- 'Elimination', because we get rid of y or 'eliminate' y from the second equation. 'Substitution', because we 'substitute' the value of y in the second equation.

Working rule : Let the two equations be

$a_1x + b_1y + c_1 = 0$ ....(1) & $a_2x + b_2y + c_2 = 0$ ....(2)

Step I : Find the value of one variable, say y, in terms of the other i.e., x from any equation, say (1).

Step II : Substitute the value of y obtained in step 1 in the other equation i.e., equation (2). This equation becomes equation in one variable x only.
Step III : Solve the equation obtained in step II to get the value of x.
Step IV : Substitute the value of x from step II to the equation obtained in step I. From this equation, we get the value of y.

➢Remark :

- In this way, we get the solution i.e. values of x and y.
- Verification is a must to check the answer.

Ex.8 Solve each of the following system of equations by eliminating x (by substitution) : (i) x + y = 7, 2x – 3y = 11 (ii) x + y = 7, 12x + 5y = 7

Sol. (i) We have ; x + y = 7 ....(1)
2x – 3y = 11 ....(2)
We shall eliminate x by substituting its value from one equation into the other. from equaton (1), we get ;
x + y = 7 ⇒ x = 7 – y
Substituting the value of x in equation (2), we get ;
2 × (7 – y) – 3y = 11 ⇒ 14 – 2y – 3y = 11
⇒ –5y = – 3 or, y = 3/5.
Now, substituting the value of y in equation (1), we get;
x + 3/5 = 7 ⇒ x = 32/5.
Hence, x = 32/5 and y = 3/5.

(ii) We have, x + y = 7 ....(1)
12x + 5y = 7 ....(2)
From equation (1), we have;
x + y = 7 ⇒ x = 7 – y
Substituting the value of y in equation (2), we get ;
12(7 – y) + 5y = 7 ⇒ 84 – 12y + 5y = 7
⇒ –7y = – 77 ⇒ y = 11
Now, Substituting the value of y in equation (1), we get ;
x + 11 = 7 ⇒ x = – 4
Hence, x = – 4, y = 11.

Ex.9 Solve the following systems of equations, $\frac{15}{u} + \frac{2}{v} = 17$ & $\frac{1}{u} + \frac{1}{v} = \frac{36}{5}$

Sol. (i) The given system of equation is ; $\frac{15}{u} + \frac{2}{v} = 17$ ....(1) $\frac{1}{u} + \frac{1}{v} = \frac{36}{5}$ ....(2)

Considering 1/u = x, 1/v = y, the above system of linear equations can be written as :

15x + 2y = 17 ....(3) $x + y = \frac{36}{5}$ ....(4)

Multiplying (4) by 15 and (iii) by 1, we get ; 15x + 2y = 17 ....(5)

$15x + 15y = \frac{36}{5} \times 15 = 108$ ....(6)

Subtracting (6) form (5), we get; –13y = – 91 ⇒ y = 7

Substituting y = 7 in (4), we get ; $x + 7 = \frac{36}{5} \Rightarrow x = \frac{36}{5} - 7 = \frac{1}{5}$

But, $y = \frac{1}{v} = 7 \Rightarrow v = \frac{1}{7}$ and, $x = \frac{1}{u} = \frac{1}{5} \Rightarrow u = 5$

Hence, the required solution of the given system is u = 5, v = 1/7.

➢ Method of Elimination By Equating the Coefficients:

Step I : Let the two equations obtained be $a_1x + b_1y + c_1 = 0$ ....(1) $a_2x + b_2y + c_2 = 0$ ....(2)
Step II : Multiplying the given equation so as to make the co-efficient of the variable to be eliminated equal.
Step III : Add or subtract the equations so obtained in Step II, as the terms having the same co-efficient may be either of opposite or the same sign.
Step IV : Solve the equations in one variable so obtained in Step III.
Step V : Substitute the value found in Step IV in any one of the given equations and then compute the value of the other variable.

Type-01. Solving Simultaneous Linear equations in Two Variables :

Ex.10 Solve the following system of equations by using the method of elimination by equating the co-efficient.

$\frac{x}{2} + \frac{2y}{5} + 2 = 10$; $\frac{2x}{7} - \frac{y}{2} + 1 = 9$

Sol. The given system of equation is $\frac{x}{2} + \frac{2y}{5} + 2 = 10 \Rightarrow \frac{x}{2} + \frac{2y}{5} = 8$ ...(1)

$\frac{2x}{5} - \frac{y}{2} + 1 = 9 \Rightarrow \frac{2x}{7} - \frac{y}{2} = 8$ ....(2)

The equation (1) can be expressed as : $\frac{5x + 4y}{10} = 8 \Rightarrow 5x + 4y = 80$ ....(3)

Similarly, the equation (2) can be expressed as : $\frac{4x-7y}{14} = 8 \Rightarrow 4x - 7y = 112$ ....(4)

Now, the new system of equations is $5x + 4y = 80$ ....(5) & $4x - 7y = 112$ ....(6)

Now multiplying equation (5) by 4 and equation (6) by 5, we get $20x - 16y = 320$ ....(7) & $20x + 35y = 560$ ....(8)

Subtracting equation (7) from (8), we get ; $y = \frac{-240}{51}$ Putting $y = \frac{-240}{51}$ in equation (5), we get ;

$5x + 4 \times \left(\frac{-240}{51}\right) = 80 \Rightarrow 5x - \frac{960}{51} = 80 \Rightarrow 5x = 80 + \frac{960}{51} = \frac{4080+960}{51} = \frac{5040}{51} \Rightarrow x = \frac{5040}{255} = \frac{1008}{51} = \frac{336}{17}$

$\Rightarrow x = \frac{336}{17}$ Hence, the solution of the system of equations is, $x = \frac{336}{17}$, $y = \frac{-80}{17}$.

Type-02. Solving a system of equations which is reduce to a system of simultaneous linear equations:

Ex.11 Solve, $\frac{2}{x+2y} + \frac{6}{2x-y} = 4$ $\frac{5}{2(x+2y)} + \frac{1}{3(2x-y)} = 1$ where, $x + 2y \neq 0$ and $2x - y \neq 0$

Sol. Taking $\frac{1}{x+2y} = u$ and $\frac{1}{2x-y} = v$, the above system of equations becomes

$2u + 6v = 4$ ....(1) $\frac{5u}{2} + \frac{v}{3} = 1$ ....(2)

Multiplying equation (2) by 18, we have; $45u + 6v = 18$ ....(3)

Now, subtracting equation (3) from equation (1), we get ; $-43u = -14 \Rightarrow u = \frac{14}{43}$

Putting u = 14/43 in equation (1), we get $2 \times \frac{14}{43} + 6v = 4 \Rightarrow 6v = 4 - \frac{28}{43} = \frac{172-28}{43} \Rightarrow v = \frac{144}{43}$

Now, $u = \frac{14}{43} = \frac{1}{x+2y} \Rightarrow 14x + 28y = 43$ ....(4)

And, $v = \frac{144}{43} = \frac{1}{2x-y} \Rightarrow 288x - 144y = 43$ ....(5)

Multiplying equation (4) by 288 and (5) by 14, the system of equations becomes

$288 \times 14x + 28y \times 288 = 43 \times 288 \Rightarrow 288x \times 14 - 144y \times 14 = 43 \times 4 \Rightarrow 4022x + 8064y = 12384$ ....(6)

$4022x - 2016y = 602$ .....(7)

Subtracting equation (7) from (6), we get; $10080y = 11782 \Rightarrow y = 1.6$(approx)

Now, putting 1.6 in (4), we get, $14x + 28 \times 1.6 = 63 \Rightarrow 14x + 44.8 = 63 \Rightarrow 14x = 18.2$

$\Rightarrow x = \frac{18.2}{14} = 1.3$ (approx) Thus, solution of the given system of equation is x = 1.3 (approx), y = 1.6 (approx)

Type-03 Equation of the form $ax + by = c$ and $bx + ay = d$ where $a \neq b$:

We may use the following method to solve the above type of equations. Steps :

Step I : Let us write the equations in the form $ax + by = c$ $bx + ay = d$

Step II : Adding and subtracting the above type of two equations, we find :

$(a + b)x + (a + b)y = c + d \Rightarrow x + y = \frac{c+d}{a+b}$ ....(1)

$(a - b)x - (a - b)y = c - d \Rightarrow x - y = \frac{c-d}{a-b}$ ....(2)

Step III : We get the values of x and y after adding or subtracting the equations (1) and (2).

Ex.12 Solve the following equations. $156x + 112y = 580$; $112x + 156y = 492$

Sol. The given system of equation is $156x + 112y = 580$ ....(1) $112x + 156y = 492$ ....(2)

Adding equation (1) and (2) we get ; $268x + 268y = 1072 \Rightarrow 268(x + y) = 1072 \Rightarrow x + y = 4$ ....(3)

Subtracting equation (2) from equation (1), we get $44x - 44y = 88 \Rightarrow x - y = 2$ ....(4)

Adding equation (3) with equation (4), we get; $2x = 6 \Rightarrow x = 3$

Putting x = 3 in equation (3), we get; y = 1

Thus, solution of the system of equations is x = 3, y = 1

Type-04. Equation of The Form, $a_1x + b_1y + c_1z = d_1$ $a_2x + b_2y + c_2z = d_2$ $a_3x + b_3y + c_3z = d_3$

We may use the following method to solve the above type of equations. Steps :

Step I : Consider any one of the three given equations.

Step II : Find the value of one of the variable, say z, from it.

Step III : Substitute the value of z found in Step II in the other two equations to get two linear equations in x, y.
Step IV : Taking the help of elimination method, solve the equations in x, y obtained in Step III.
Step V : Substitute the values of x, y found in Step IV and Step II to get the value of z.

Ex.13 Solve, $x + 2y + z = 12$
$2x - z = 4$
$x - 2y = 4$

Sol. We have, $x + 2y + z = 12$ ....(1) $2x - z = 4$ ....(2) $x - 2y = 4$ ....(3)
From equation (1), we have $z = 12 - x - 2y$.
Now Putting, $z = 12 - x - 2y$ in the equation (2), we get;
$2x - (12 - x - 2y) = 4 \Rightarrow 2x - 12 + x + 2y = 4 \Rightarrow 3x + 2y = 16$ ....(4)
Adding equations (3) and (4), we get; $4x = 20 \Rightarrow x = 5$
Putting the value of x = 5 in equation (2), we get $2 \times 5 - z = 4 \Rightarrow z = 10 - 4 = 6$
Again putting the value of x = 5 in equation (3), we get $5 - 2y = 4 \Rightarrow y = 1/2$
Hence, the solution of the given system of equations is ; $x = 5,\ y = 1/2,\ z = 6$

➢ Cross-Multiplication Method :

By the method of elimination by substitution, only those equations can be solved, which have unique solution. But the method of cross multiplication discussed below is applicable in all the cases; whether the system has a unique solution, no solution or infinitely many solutions.
Let us solve the following system of equations $a_1x + b_1y + c_1 = 0$ ....(1) $a_2x + b_2y + c_2 = 0$ ....(2)
Multiplying equation (1) by b_2 and equation (2) by b_1, we get
$a_1b_2x + b_1b_2y + b_2c_1 = 0$ ....(3) $a_2b_1x + b_1b_2y + b_1c_2 = 0$ ....(4)
Subtracting equation (4) from equation (3), we get

$$(a_1b_2 - a_2b_1)\,x + (b_2c_1 - b_1c_2) = 0 \Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \left[\begin{matrix} a_1b_2 - a_2b_1 \neq 0 \\ \dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2} \end{matrix}\right]$$

Similarly, $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$ These values of x and y can also be written as: $\dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{-y}{a_1c_2 - a_2c_1} = \dfrac{1}{a_1b_2 - a_2b_1}$

Ex.14 Solve the following system of equations by cross-multiplication method. $2x + 3y + 8 = 0$ & $4x + 5y + 14 = 0$
Sol. The given system of equations is $2x + 3y + 8 = 0$ & $4x + 5y + 14 = 0$
By cross-multiplication, we get

$$\frac{x}{\begin{matrix}3 & 8\\ 5 & 14\end{matrix}} = \frac{-y}{\begin{matrix}2 & 8\\ 4 & 14\end{matrix}} = \frac{1}{\begin{matrix}2 & 3\\ 4 & 5\end{matrix}} \Rightarrow \frac{x}{3\times14 - 5\times8} = \frac{-y}{2\times14 - 4\times8} = \frac{1}{2\times5 - 4\times3} \Rightarrow \frac{x}{42-40} = \frac{-y}{28-32} = \frac{1}{10-12}$$

$$\Rightarrow \frac{x}{2} = \frac{-y}{-4} = \frac{1}{-2} \Rightarrow \frac{x}{2} = -\frac{1}{2} \Rightarrow x = -1 \quad \text{and} \quad \frac{-y}{-4} = -\frac{1}{2} \Rightarrow y = -2.$$

Hence, the solution is $x = -1,\ y = -2$ We can verify the solution.

✍Homogeneous equations :

The system of equations $a_1x + b_1y = 0$ & $a_2x + b_2y = 0$
called homogeneous equations has only solution x = 0, y = 0, when $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

(i) when $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$, The system of equations has only one solution, and the system is consistent.

(ii) When $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$ The system of equations has infinitely many solutions and the system is consistent.

Ex.15 Find the value of k for which the system of equations $4x + 5y = 0$; $kx + 10y = 0$ has infinitely many solutions.
Sol. The given system is of the form $a_1x + b_1y = 0$ & $a_2x + b_2y = 0$
$a_1 = 4,\ b_1 = 5$ and $a_2 = k,\ b_2 = 10$
If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$, the system has infinitely many solutions. $\Rightarrow \dfrac{4}{k} = \dfrac{5}{10} \Rightarrow k = 8$

✍Word Problems on Simultaneous linear equation :

01. Problems Based on Articles :

Ex.16. The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, he buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.
Sol. Let the cost of one bat be ₹ x and cost of one ball be ₹ y. Then

$7x + 6y = 3800$ ....(1) $\qquad 3x + 5y = 1750$ ....(2)

From (1) $y = \frac{3800 - 7x}{6}$ $\qquad$ Putting $y = \frac{3800 - 7x}{6}$ in (2), we get $\quad 3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$ ....(3)

Multiplying (3) by 6, we get $\quad 18x + 5(3800 - 7x) = 10500 \Rightarrow 18x + 19000 - 35x = 10500$

$\Rightarrow -17x = 10500 - 19000 \quad \Rightarrow -17x = -8500 \quad \Rightarrow x = 500$

Putting x = 500 in (1), we get $\quad 7(500) + 6y = 3800 \Rightarrow 3500 + 6y = 3800 \Rightarrow 6y = 3800 - 3500$

$\Rightarrow 6y = 300 \quad \Rightarrow y = 50$

Hence, the cost of one bat = ₹ 500 and the cost of one ball = ₹ 50

02. Problems Based on Numbers :

Ex.17. What number must be added to each of the numbers, 5, 9, 17, 27 to make the numbers in proportion ?

Sol. Four numbers are in proportion if $\quad$ First × Fourth = Second × Third.

Let x be added to each of the given numbers to make the numbers in proportion. Then,

$(5 + x)(27 + x) = (9 + x)(17 + x) \Rightarrow 135 + 32x + x^2 = 153 + 26x + x^2 \Rightarrow 32x - 26x = 153 - 135$

$\Rightarrow 6x = 18 \quad \Rightarrow x = 3$

03. Problems Based on Ages :

Ex.18 Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

Sol. Let the age of father = x years. And the sum of the ages of his two children = y years

According to the question, Father's age = 3 × (sum of the ages of his two children) $\Rightarrow x = 3y$ ....(1)

After 5 years, Father's age = (x + 5) years &

sum of the ages of his two children's = y + 5 + 5 = y + 10 $\qquad$ [Age of his each children increases by 5 years]

According to the question, After 5 years

Father's age = 2 × (sum of ages of his two children) $\Rightarrow x + 5 = 2 \times (y + 10) \Rightarrow x + 5 = 2y + 20 \Rightarrow x - 2y = 15$ ....(2)

Putting x = 3y from (1) in (2), we get $\quad 3y - 2y = 15 \Rightarrow y = 15$ years And $x = 3y \Rightarrow x = 3 \times 15 = 45$

$\Rightarrow x = 45$ years. $\qquad$ Hence, father's age = 45 years

04. Problems Based on two digit numbers:

Ex.19 The sum of a two digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number.

Sol. Let the unit's place digit be x and the ten's place digit be y.

$\therefore$ Original number = x + 10 y $\qquad$ The number obtained by reversing the digits = 10x + y

According to the question, Original number + Reversed number = 99

$\Rightarrow (x + 10y) + (10x + y) = 99 \Rightarrow 11x + 11y = 99 \Rightarrow x + y = 9 \Rightarrow x = 9 - y$ ....(1)

Given the difference of the digit = 3 $\Rightarrow x - y = 3$ ....(2)

On putting the value of x = 9 – y from equation (1) in equation (2), we get

$(9 - y) - y = 3 \Rightarrow 9 - 2y = 3 \Rightarrow 2y = 6 \Rightarrow y = 3$

Substituting the value of y = 3 in equation (1), we get $\quad x = 9 - y = 9 - 3 = 6$

Hence, the number is $\quad x + 10y = 6 + 10 \times 3 = 36$.

05. Problems Based on Fraction :

Ex.20 The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

Sol. Let Numerator = x and Denominator = y $\qquad \therefore$ Fraction = $\frac{x}{y}$

According to the first condition, Numerator + denominator = 2 × numerator + 4

$\Rightarrow x + y = 2x + 4 \Rightarrow y = x + 4$ ....(1)

According to the second condition, $\frac{\text{Increased numerator by 3}}{\text{Increased denominator by 3}} = \frac{2}{3} \Rightarrow \frac{x+3}{y+3} = \frac{2}{3}$

$\Rightarrow 3x + 9 = 2y + 6 \Rightarrow 3x - 2y + 3 = 0$ ....(2)

Substituting the value of y form equation (1) into equation (2), we get

$3x - 2(x + 4) + 3 = 0 \Rightarrow 3x - 2x - 8 + 3 = 0 \Rightarrow x = 5$

On putting x = 5 in equation (1), we get $\quad y = 5 + 4 \Rightarrow y = 9 \qquad$ Hence, the fraction $= \frac{x}{y} = \frac{5}{9}$

❖❖❖

☎ आपका परिश्रम + हमारा मार्गदर्शन = निश्चित सफलता ☎

❖❖❖

CBSE

Linear Equation in Two Variables

For Class - IX

Chemistry By

Er. Jitendra Gupta sir

Marks = 15 Date : 20/05/2020 Time = 45 Min.

General Instructions-

- Every question is compulsory.
- Keep answer copy neat and clean.
- use of calculator, slide rule, graph paper & trigonometric tables is Not Permitted.

Section-'A' Each Questions-1 Mark

1. What types of lines do the pair of equations x=c and y=c represent graphically?
2. A boat is moving at the rate of 5km/h in still water, takes thrice as much as time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.
3. Find the value of m, when (m+1)x=3ky+15=0 and 5x+ky+5=0 are coincident.
4. Write the pair of linear equations which have solutions x=2, y=-2.
5. Solve it on a graph 4x-3y+4=0, 4x+3y-24=0.

Section-'B' Each Questions 2-Mark.

6. If we have two variables x and y when x=a and y=b is the solution of equations x-y=2 and x+y=4, then what will be the value of a and b.
7. Use cross multiplication method to solve ax + bx = a-b, bx - ay = a+b.
8. Whether this pair of linear equations is consistent. Find x-2y=6, 3x-6y=0.
9. A number is a two digit number which is three times more than 4 times the sum of the digits. If 18 is added to the number, the digits gets opposite. Represent geometrically.
10. The addition of numerator and denominator of a fraction is three less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Find the fraction.

High Order Thinking (HOT) Problems

(For Home Work ONLY)

1. 6 men and 10 women can finish making pots in 8 days, while the 4 men and 6 women can finish it in 12 days. Find the time taken by the one man alone from that of one woman alone to finish the work.
2. A boat covers 14 kms in upstream and 20 kms downstream in 7 hours. Also it covers 22 kms upstream and 34 kms downstream in 10 hours. Find the speed of the boat in still water and of that the stream.
3. Draw the graph of 2x+y=6 and 2x-y+2=0. Shade the region bounded by these lines and x axis. Find the area of the shaded region
4. When you add two numbers and the number obtained by reversing the order of its digits is 165. If the both numbers differ by three, find the number.
5. A number say z is exactly the four times the sum of its digits and twice the product of the digits. Find the numbers.
6. Solve graphically 4x-3y+4=0, 4x+3y-20=0
7. There are two points on a highway a,b. They are 70 km apart. An auto starts from A and another auto starts from B simultaneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other they meet in 1 hour. Find how fast the two autos are.
8. A diver rowing at the rate of 5 km/h in still water takes double the time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.
9. The larger of two supplementary angles exceeds thrice the smaller by 20 degrees. Find them.
10. The sum of two children is ‘a’. The age of the father is twice the ‘a’. After twenty years, his age will be equal to the addition of the ages of his children. Find the age of father.

***** With Best Wishes *****